

Risk-adjusted expected return for selection decisions¹

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ABSTRACT: The results of genetic evaluation are predictions of breeding values for the selection candidates, and these involve uncertainty with regard to future returns from the use of those selected individuals. This uncertainty is due to differential variability in BLUP of breeding values and can be translated into risk: High fluctuations mean greater risk, which is not taken into account by just looking at expected return. In this research, the methodology of value at risk (VaR) and expected shortfall is introduced for animal breeding decisions as a means to adjust the expected return for the cost of uncertainty in prediction of breeding values. This methodology has recently received a great deal of attention from financial institutions. Given a specified probability α , VaR is the α -quantile of the distribution of economic returns. The conditional value at risk (CVaR), or expected shortfall, is the expectation of the

economic returns, which are less or equal to the VaR. By subtracting the CVaR from the predicted aggregate genotype (μ_R), a risk-adjusted expected return (RAER) measure was obtained. The measures μ_R , VaR, and RAER were calculated for a data set with progeny of 161 Polled Hereford bulls belonging to a beef cattle company. The Pearson and Spearman correlations between μ_R and RAER were 0.89 ($P < 0.001$) and 0.90 ($P < 0.001$), respectively. Even though the latter correlation was high, some bulls ranked differently for μ_R compared with RAER. The Pearson correlation between μ_R and VaR was low (0.124) and nonsignificant ($P > 0.05$), whereas the correlation between VaR and RAER was -0.31 ($P < 0.0001$). The results indicate the need to take into account the adjustment for risk in expected return in order to alleviate the effects of possible losses when overrated animals are selected.

Key Words: Errors, Genetic Improvement, Prediction, Returns, Risk

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Introduction

Most selection decisions in animal breeding are based on several traits, being the concept of the aggregate genotype of Hazel (1943) the most grounded approach to a multiple-trait selection criterion. The predicted H is a combination of BLUP (Henderson, 1984) of breeding values (**BV**) for the selection candidates weighted by the economic values. Therefore, uncertainty in future outcomes may result from inaccuracies in the estimation of either the economic values or the BLUP(BV), and can be translated into risk: High fluctuations mean greater uncertainty, thus greater risk. Recently, Kulak

et al. (2003) observed that “risk can make a large difference in economic values, but may have a smaller effect on ratios of economic weights, and on the magnitude and direction of genetic change.” In the present research, we will explore the uncertainty in future returns due to variability in BLUP(BV). Sires with higher prediction error variances (**PEV**) are riskier to use than sires with lower PEV (Dematawewa et al., 1998). Additionally, we calculate the predicted aggregate genotype conditional on a fixed set of economic values as in the classical approach (Hazel, 1943).

The characterization of the distribution of future returns R is the key element of modern financial risk management theory (Diebold et al., 1998). Given a probability α , value at risk (**VaR**; Jorion, 1997) is the negative α -quantile of R . A potential use of VaR methodology in animal breeding is the assessment of risk brought about by breeding decisions. Artzner et al. (1999) proposed another measure of risk, the conditional value-at-risk (**CVaR**) or expected shortfall, which is equal to the expectation of those values of R that are less than or equal to VaR. The goal of this research is to assess the use of VaR and CVaR as a means to incorporate

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risk into breeding decisions, in order to account for the uncertainty in expected returns due to differential PEV of BLUP(BV).

Materials and Methods

Value at Risk

Given a specified small probability α , VaR is the α -quantile of the distribution of economic returns R . More formally, let R be a random variable with cumulative distribution function $F(R)$, and let VaR be a fixed value of R such that:

$$\alpha = \Pr (R \leq \text{VaR}) = F_R(\text{VaR})$$

Then, on using the inverse function of the cumulative distribution function, VaR is equal to:

$$\text{VaR} = F_R^{-1}(\alpha) \tag{1}$$

We will require the distribution of R to be continuous. When R is normally distributed with mean μ_R and variance σ_R^2 , its standardized value is equal to:

$$R^* = \frac{R - \mu_R}{\sigma_R} \tag{2}$$

The approach to calculate VaR used here is parametric and requires estimating μ_R and σ_R^2 . Denoting with VaR* the value of the standardized normal distribution of R (R^*) that corresponds to the α -quantile as in [1], VaR is equal to:

$$\text{VaR} = |\text{VaR}^*| \sigma_R \tag{3}$$

Note that VaR is equal to the product of the absolute value of the standard normal density at the associated probability α of observing a loss, by the standard deviation of R .

Conditional VaR or Expected Shortfall

The CVaR is the expected value of the distribution of R , conditional on R being less than or equal to VaR:

$$\text{CVaR} = E(R|R \leq \text{VaR}) \tag{4}$$

When $R \sim N(0,1)$, we use the notation CVaR* to refer to a number equal to the selection intensity for a probability of truncation selection equal to α . For example, for $\alpha = 0.05$, CVaR* = 2.063. Rockafellar and Uryasev (2000) showed that, under normality, the use of VaR is numerically equivalent to using CVaR in [4].

Risk-Adjusted Expected Return

After expression [1] in Sharpe (1991), we define risk-adjusted expected return (**RAER**) for animal i as follows:

$$\text{RAER}_i = \mu_{R_i} - \frac{\sigma_{R_i}^2}{\tau} \tag{5}$$

where the parameter τ is the tolerance for risk (or Sharpe's ratio) of the manager that makes the breeding decision. Sharpe (1991) interpreted τ as an "investor's marginal rate of substitution of variance for expected return." After Rockafellar and Uryasev (2000), and for normally distributed R , we propose to take τ as the reciprocal of the product of the standard deviation of R by the CVaR*, so that for $\alpha = 0.05$ the RAER is equal to:

$$\text{RAER}_i = \mu_{R_i} - 2.063\sigma_{R_i} \tag{6}$$

To frame Expression [6] in a context of animal breeding, we define the expectation of R for animal i , conditional on a fixed set of economic values, to be equal to the predicted aggregate genotype (Hazel, 1943):

$$\mu_{R_i} = E(R_i|\mathbf{y}) = \mathbf{e}'E(\mathbf{a}_i|\mathbf{y}) = \mathbf{e}'\hat{\mathbf{a}}_i \tag{7}$$

The vector \mathbf{e} includes the economic values for all animals and the vector $\hat{\mathbf{a}}_i$ is the BLUP(BV) for animal i . Predictions were assumed to be distributed in a multivariate normal fashion. The standard deviation of R_i is calculated as the square root of its PEV, which is equal to:

$$\sigma_{R_i}^2 = \text{Var}(R_i|\hat{R}_i) = \mathbf{e}' \text{Var}(\hat{\mathbf{a}}_i - \mathbf{a}_i)\mathbf{e} = \mathbf{e}'\mathbf{C}_i^{aa}\mathbf{e} \tag{8}$$

where $\text{Var}(\hat{\mathbf{a}}_i - \mathbf{a}_i)$ is the PEV of the individual's BV in the vector \mathbf{a}_i , and is equal to the submatrix (\mathbf{C}_i^{aa}) of the inverse of the coefficient matrix in the 'mixed model equations' corresponding to \mathbf{a}_i . To obtain RAER_{*i*}, we replaced with Expression [7] and the square root of Expression [8] into Expression [6] to obtain:

$$\text{RAER}_i = \mathbf{e}'\hat{\mathbf{a}}_i - 2.063\sqrt{\mathbf{e}'\mathbf{C}_i^{aa}\mathbf{e}} \tag{9}$$

The animals with maximal values of [9] maximize expected return while taking into account the average of all possible extreme losses in the tail of the distribution.

Data

Data used to exemplify the use of the VaR measures were predictions of breeding values for 161 Polled Hereford bulls that sired at least one calf with records of birth and weaning weight. The animals belong to a ranch of Las Lilas cattle company located in Pasteur, a province of Buenos Aires, Argentina. The company has its own genetic evaluation system developed by personnel of the college of agriculture from Universidad de Buenos Aires. Data collected from 1972 to 2000 were 5,354 birth weights (**BiW**), 13,111 weaning weights (**WW**), and 5,881 weights at 15 mo (**FW**). The average age at weaning and at 15 mo was 197 and 472 d, respec-

Table 1. Descriptive statistics for all traits and corresponding economic values

Trait	Birth weight	Weaning weight	Maternal weaning weight	15-mo weight
Number of records	5,354	13,111	—	5,881
Phenotypic mean, kg	392.0	191.9	—	420.4
Additive standard deviation, kg	3.32	18.80	18.80	24.09
Number of discounted expressions	456.0	423.4	267.6	185.3
Absolute economic value, \$/yr	-3,807.5	5,169.9	3,264.6	2,692.2

tively. A summary of descriptive statistics is included in Table 1. A multiple trait additive animal model was employed to obtain BLUP(BV) for BiW, WW, maternal WW (MW), and FW. The covariance components were estimated by REML under the Expectation Maximization algorithm. The complete mixed model equation matrix was inverted to calculate the PEV of sire i as C_i^{aa} . Positive genetic trends were estimated across all traits. The range of BLUP(BV) (not EPD) were -4.63 to 7.43 kg for BW; -18.44 to +54.38 for WW, -21.52 to +23.89 for MW, and -44.97 to 80.25 kg for FW. Corresponding accuracies were 0.33 to 0.97, 0.27 to 0.97, 0.11 to 0.96 and 0.28 to 0.97 for BW, WW, MW, and FW, respectively. The accuracy for trait j of animal i was calculated as follows (Mrode, 1996):

$$[(g_j - PEV_{ij})/g_j]^{1/2}$$

where g_j is the additive variance component for trait j .

Economic Values

In order to obtain the economic values for BW, WW, MW, and FW, a profit function for a pasture based beef production system in Argentina was developed using the methodology proposed by Ponzone and Newman (1989). The discounted expressions were calculated for a herd of 100 cows and a period of 20 yr, as described by Brascamp (1978). The absolute expressions of the economic values were obtained by multiplying the derivatives of the profit function by the total number of discounted expressions for each trait. Discounted expressions and economic values are displayed in Table 1.

Results

An Example

In order to illustrate the calculation of the VaR and RAER measures, the results for bull 2880 are shown in detail. The BLUP of BW, WW, MW, and FW in kilograms, were respectively equal to 3.95, 52.99, -19.49, and 80.25. The matrix of genetic (co)variance components and the PEV matrix for bull 2880 are respectively equal to:

$$\begin{bmatrix} 11.035 & 22.706 & 0.000 & 42.416 \\ 22.706 & 206.579 & -57.302 & 216.573 \\ 0.000 & -57.302 & 204.289 & 0.000 \\ 42.416 & 216.573 & 0.000 & 580.172 \end{bmatrix}$$

$$\begin{bmatrix} 0.987 & 1.652 & 0.006 & 2.909 \\ 1.652 & 29.130 & -9.286 & 25.861 \\ 0.006 & -9.286 & 61.560 & -0.440 \\ 2.909 & 25.861 & -0.440 & 84.739 \end{bmatrix}$$

so that the accuracies for BW, WW, MW, and FW were equal to 0.95, 0.93, 0.84, and 0.92, respectively. The predicted aggregate genotype was \$411,337, and its standard deviation (calculated as the square root of Expression [8]) was \$48,342.70. Thus, for $\alpha = 0.05$, $VaR_{2880} = 1.645 (\$48,342.70) = \$79,523.70$ and $CVaR_{2880} = 2.063 (\$48,342.70) = \$99,731$ and $RAER_{2880} = \$411,337 - \$99,731 = \$311,606$.

Correlations Among Return, Risk-Adjusted Expected Return and Value at Risk

Table 2 displays the values of μ_{Ri} , RAER, and VaR for representative bulls from three distinctive groups of sires. Bulls 2880 and 3164 ranked highest for both μ_{Ri} and RAER. At the other extreme, bulls 2219 and 1795 had the lowest values of both R and RAER. The Pearson correlation between μ_{Ri} and RAER was 0.89, whereas the Spearman correlation among the ranks of both measures was equal to 0.90. Even though the latter correlation was high ($P < 0.001$), there was some re-ranking for RAER compared with μ_{Ri} . Thus, 16 out of the 25 bulls that ranked on the top of μ_{Ri} were also within the top-ranked 25 sires for RAER. In this respect, Table 2 displays two extreme cases worth noting. Sire 2247, which ranked in position 113 for μ_{Ri} , reached position 58 for RAER. On the other hand, sire 3375, in position 61 for μ_{Ri} , fell 45 places to position 106 for RAER. This is due to a low risk adjustment for sire 2247 ($\sigma_R = 28,375$) compared with a high adjustment for bull 3375 ($\sigma_R = 113,286$). The Pearson correlation between μ_{Ri} and VaR was low (0.124) and nonsignificant ($P > 0.05$), whereas the one between VaR and RAER was -0.31 ($P < 0.0001$).

Table 2. Return, standard deviation of return, risk-adjusted expected return and value at risk for a subset of Polled Hereford bulls

Identification of sire	Expected return, \$	Standard deviation of return, \$	Risk-adjusted expected return, \$	Value at risk, \$
2880	411,337	48,343	311,606	79,524
3164	390,982	75,991	234,214	125,005
2247	12,097	28,375	-46,440	46,677
3375	91,006	113,286	-142,702	186,355
2219	-162,562	75,110	-317,514	123,556
1795	-220,910	58,838	-342,293	96,789
Averages	64,390	73,851	-87,964	121,485

Discussion

The main contribution of the present research was to introduce the VaR and expected shortfall measures for animal selection decisions. The VaR measures financial risk and consists of calculating the lower percentile of the distribution of the economic return for the use of a sire or of a dam. Thus, it not only reflects risk exposure but also the probability of financial loss (Jorion, 1997). The VaR is a function of the prediction error (co)variances for each animal and economic weights, expressed in terms of money. Jorion (1997) discussed the time horizon for which the risk measures apply in finance, and this ranges in days or months. By contrast, animal breeding applications require longer time horizons because returns are realized when progeny and further descendants are sold. Since the period chosen in the present research is 20 yr, a loss of R equal to or greater than VaR will be observed with probability of 5% in 20 yr, assuming normality of the distribution of returns. The sire with the highest risk had a VaR of \$190,507, which means \$9,525.40/yr, or \$95.25/cow. On the other extreme, the bull with the lowest VaR had a 5% chance of incurring a loss equal to or greater than \$46,677 (\$2,333.85/yr, or \$23.33/cow). Manfredo and Leuthold (1999) indicated that there are three classical approaches to calculating VaR: the historical simulation method, the variance-covariance approach, and Monte Carlo simulation. The one used here is the parametric variance-covariance approach, also known as the delta-normal method (Jorion, 1997). This approach was selected because the aggregate genotype is a linear combination of normal random variables. The two other methods (historical simulation and Monte Carlo simulation) require the availability of time series of prices for the different selection candidates.

Selecting exclusively on the aggregate genotype implicitly assumes a risk free environment. For example, when two sires have the same EPD for all traits, both animals will have the same aggregate genotype irrespective of the level of accuracy of those EPD (Dematawewa et al., 1998). The proposed risk-adjusted expected return involves both aggregate genotype and the cost of the risk associated with the uncertainty in predicting aggregate genotype with its BLUP (i.e., μ_{Ri}). The adjust-

ment is performed by subtracting from μ_{Ri} the expected shortfall. Therefore, RAER allows for expanding the selection criterion to take a downside risk into account. Downside risk refers to the chance of losing R from selecting animals with lower aggregate genotype than predicted, and focuses on the downside tail of the distribution of returns. The idea that risk should be involved in selection decisions was proposed by Schneeberger et al. (1982) and further considered by Klieve et al. (1993) and Dematawewa et al. (1998). Schneeberger et al. (1982) introduced the idea of expected return and risk in dairy bull selection, and used a utility function under the mean-variance approach set forth by H. Markowitz in 1959 (see Markowitz, 1991). The RAER measure attaches a negative weight to losing money from a selection decision with uncertain future outcome, due to increased prediction error (co)variances. This negative weight consists of the average of all possible extreme losses in the tail of the distribution. Klieve et al. (1993) assessed the cost of selecting animals with either high or low accuracy in terms of the loss in genetic response. Also, Amer and Hofer (1994) observed that benefits from genetic improvement should account for uncertainty in the value of selection. In the present research, the source of uncertainty is due to having to predict breeding values, and money will be lost when overrated animals are selected.

As in the current research, Schneeberger et al. (1982) and Dematawewa et al. (1998) also used PEV to estimate the variance of R (Expression [8]). The impact of estimating VaR, either with the complete PEV matrix for each selection candidate or with a diagonal PEV (i.e., covariance of prediction errors assumed to be zero), was considered by Pruzzo et al. (2002). The differences in VaR calculated with either method increased with small accuracies. These latter values are usually calculated by approximating diagonal elements of the PEV matrix (Harris and Johnson, 1998). As the inversion of the coefficient matrix of the mixed model equations is generally prohibitive, both diagonal and off-diagonal elements of PEV are not usually available. However, García-Cortés et al. (1992) and Fouilloux and Laloë (2001) proposed sampling methods to calculate both diagonal and off-diagonal elements of PEV, which can be used for the endeavor.

Implications

Differential variability in predictions of breeding values can be translated into risk: High fluctuations mean greater uncertainty, thus greater risk. Therefore, animals with the same predicted values and different accuracies for all traits will have the same predicted aggregate genotype. However, those animals with lower accuracies are riskier to use than animals with higher accuracies, and this is not taken into account by just looking at expected return. We applied the methodology of value at risk and expected shortfall to adjust the expected return for the cost of uncertainty in prediction of breeding values. In doing so, key concepts in modern financial risk management theory have been included into breeding decisions.

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